International Journal of Scientific & Engineering Research, Volume 4, Issue 8 ISSN 2229-5518 1

Cordial Labeling of Fan Related Graphs

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Abstract—In this paper we prove that the graph obtained by joining two copies of fan graph ($F_n = P_n + K_1$) by a path of arbitrary length is cordial. Further we prove that the star of fan graph is cordial and the graph obtained by joining two copies of star of fan graph by a path of arbitrary length is cordial.

Index Terms—Cordial graph, Fan graph, Star of a graph.

1 Introduction

We consider simple, finite, undirected graph $G = (V, E)$. In this paper F_n denotes fan graph with $n+1$ vertices and F_n^* denotes the star of fan graph. For all other terminology and notations we follow Harary[1]. First we will provide some useful definitions for the present investigations.

Definition 1.1 A *star graph* with n vertices is a tree with one vertex having degree $n-1$ and other $n-1$ vertices having degree 1.

Definition 1.2 A *star of a graph* is obtained by replacing each vertex of star graph $K_{1,n}$ by a graph G. We denote it as G^*.

Definition 1.3 If the vertices of the graph are assigned values subject to certain conditions is known as *graph labeling*.
A survey on graph labeling is given by Gallian[2].

Definition 1.4 Let $G = (V, E)$ be a graph. A mapping $f : V(G) \rightarrow \{0, 1\}$ is called *binary vertex labeling* of G and $f(v)$ is called *label of the vertex* v of G under f.

For an edge $e = uv$, the induced edge labeling $f^* : E(G) \rightarrow \{0, 1\}$ is given by $f^*(e) = |f(u) - f(v)|$. Let us denote the number of vertices of G having labels 0, 1 by $v_f(0)$, $v_f(1)$ respectively under f and the number of edges of G having labels 0, 1 by $e_f(0)$, $e_f(1)$ respectively under f^*.

Definition 1.5 A binary vertex labeling of a graph G is called *cordial labeling* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is *cordial* if it admits cordial labeling.

The concept of cordial graphs was introduced by Cahit[3].

Shee and Ho[4] proved that path union of cycles, petersen graphs, trees, wheels, unicyclic graphs are cordial.

Vaidya et al.[5] proved that the graph obtained by joining two copies of cycles by a path of arbitrary length is cordial. In [6], they proved that the graph obtained by joining two copies of petersen graph by a path of arbitrary length is cordial. In [7], the same authors proved that path union of cycle with one chord is cordial. Seuod and Abdel Maqsoud[8] proved that P_n^2 is cordial for all n. Ho et al.[9] proved that cartesian product of two cordial graphs of even size is cordial.

In the present investigations we prove that the graph obtained by joining two copies of fan graph F_n by a path of arbitrary length is cordial. We also prove that star of fan graph F_n^* is cordial. In addition to this we prove that the graph obtained by joining two copies of star of fan by a path of arbitrary length is cordial.

2 Main Results

Theorem 2.1 The graph obtained by joining two copies of fan graph F_n by a path of arbitrary length is cordial.

Proof: Let G be the graph obtained by joining two copies of fan graph $F_n = P_n + K_1$ by path P_k. Let us denote the successive vertices of first copy of fan graph by $u_1, u_2, \ldots, u_{n+1}$ (where u_1 is apex vertex) and the successive vertices of second copy of fan graph by $v_1, v_2, \ldots, v_{n+1}$ (where v_1 is apex vertex). Let $w_1, w_2, \ldots, w_k$ be the vertices of path P_k with $w_1 = u_1$ and $w_k = v_1$. Here note that for $n = 2$, F_2 is a cycle C_3 and it is already proved by Vaidya et al.[5] that the

graph obtained by joining two copies of cycles by a path of arbitrary length is cordial. Hence we consider the case for $n \geq 3$.
We define labeling function $f : V(G) \rightarrow \{0, 1\}$ as follows.

Case 1: $n \equiv 0(mod4)$.
Subcase I: $k \equiv 0(mod4)$.
$f(u_i) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $i \equiv 2, 3(mod4)$, $1 \leq i \leq n + 1$.
$f(v_i) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $i \equiv 2, 3(mod4)$, $1 \leq i \leq n + 1$.
$f(w_j) = 0$; if $j \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4)$, $1 \leq j \leq k$.

Subcase II: $k \equiv 1(mod4)$.
$f(u_i) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $i \equiv 2, 3(mod4)$, $1 \leq i \leq n + 1$.
$f(v_i) = 0$; if $i \equiv 2, 3(mod4)$
$= 1$; if $i \equiv 0, 1(mod4)$, $1 \leq i \leq n + 1$.
$f(w_k) = 1$;
$f(w_j) = 0$; if $j \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4)$, $1 \leq j \leq k - 1$.

Subcase III : $k \equiv 2(mod4)$.
$f(u_i) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $i \equiv 2, 3(mod4)$, $1 \leq i \leq n + 1$.
$f(v_i) = 0$; if $i \equiv 2, 3(mod4)$
$= 1$; if $i \equiv 0, 1(mod4)$, $1 \leq i \leq n + 1$.
$f(w_j) = 0$; if $j \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4)$, $1 \leq j \leq k$.

Subcase IV: $k \equiv 3(mod4)$.
$f(u_i) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $i \equiv 2, 3(mod4)$, $1 \leq i \leq n + 1$.
$f(v_i) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $i \equiv 2, 3(mod4)$, $1 \leq i \leq n + 1$.
$f(w_k) = 0$;
$f(w_j) = 0$; if $j \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4)$, $1 \leq j \leq k - 1$.

Case 2: $n \equiv 1(mod4)$.
Subcase I: $k \equiv 0(mod4)$.
$f(u_i) = 0$; if $i \equiv 0, 3(mod4)$
$= 1$; if $i \equiv 1, 2(mod4)$, $1 \leq i \leq n + 1$.
$f(v_i) = 0$; if $i \equiv 2, 3(mod4)$
$= 1$; if $i \equiv 0, 1(mod4)$, $1 \leq i \leq n + 1$.
$f(w_j) = 0$; if $j \equiv 2, 3(mod4)$
$= 1$; if $j \equiv 0, 1(mod4)$, $1 \leq j \leq k$.

Subcase II: $k \equiv 1(mod4)$.
$f(u_i) = 0$; if $i \equiv 0, 3(mod4)$
$= 1$; if $i \equiv 1, 2(mod4)$, $1 \leq i \leq n + 1$.
$f(v_i) = 0$; if $i \equiv 2, 3(mod4)$
$= 1$; if $i \equiv 0, 1(mod4)$, $1 \leq i \leq n + 1$.
$f(w_j) = 0$; if $j \equiv 2, 3(mod4)$
$= 1$; if $j \equiv 0, 1(mod4)$, $1 \leq j \leq k$.

Subcase III: $k \equiv 2(mod4)$.
$f(u_i) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $i \equiv 2, 3(mod4)$, $1 \leq i \leq n + 1$.
$f(v_i) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $i \equiv 2, 3(mod4)$, $1 \leq i \leq n + 1$.
$f(w_j) = 0$; if $j \equiv 1, 2(mod4)$
$= 1$; if $j \equiv 0, 3(mod4)$, $1 \leq j \leq k$.

Subcase IV: $k \equiv 3(mod4)$.
$f(u_i) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $i \equiv 2, 3(mod4)$, $1 \leq i \leq n + 1$.
$f(v_i) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $i \equiv 2, 3(mod4)$, $1 \leq i \leq n + 1$.
$f(w_k) = 0$;
$f(w_j) = 0$; if $j \equiv 1, 2(mod4)$
$= 1$; if $j \equiv 0, 3(mod4)$, $1 \leq j \leq k - 1$.

Case 3 $n \equiv 2(mod4)$.
Subcase I: $k \equiv 0(mod4)$.
$f(u_i) = 0$; if $i \equiv 2, 3(mod4)$
$= 1$; if $i \equiv 0, 1(mod4)$, $1 \leq i \leq n + 1$.
$f(v_i) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $i \equiv 2, 3(mod4)$, $1 \leq i \leq n + 1$.
$f(w_j) = 0$; if $j \equiv 0, 3(mod4)$
$= 1$; if $j \equiv 1, 2(mod4)$, $1 \leq j \leq k$.

Subcase II: $k \equiv 1(mod4)$.
$f(u_i) = 0$; if $i \equiv 2, 3(mod4)$
$= 1$; if $i \equiv 0, 1(mod4)$, $1 \leq i \leq n + 1$.
$f(v_i) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $i \equiv 2, 3(mod4)$, $1 \leq i \leq n + 1$.
$f(w_k) = 0$;
$f(w_j) = 0$; if $j \equiv 0, 3(mod4)$
$= 1$; if $j \equiv 1, 2(mod4)$, $1 \leq j \leq k - 1$.

Subcase III: $k \equiv 2(mod4)$.
$f(u_i) = 0$; if $i \equiv 2, 3(mod4)$
$= 1$; if $i \equiv 0, 1(mod4)$, $1 \leq i \leq n + 1$.
$f(v_i) = 0$; if $i \equiv 0, 3(mod4)$
$= 1$; if $i \equiv 1, 2(mod4)$, $1 \leq i \leq n + 1$.
$f(w_k) = 1$;
$f(w_j) = 0$; if $j \equiv 2, 3(mod4)$
$= 1$; if $j \equiv 0, 1(mod4)$, $1 \leq j \leq k - 1$.

Subcase IV: $k \equiv 3(mod4)$.
$f(u_i) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $i \equiv 2, 3(mod4)$, $1 \leq i \leq n + 1$.
$f(v_i) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $i \equiv 2, 3(mod4)$, $1 \leq i \leq n + 1$.
$f(w_k) = 0$;
$f(w_j) = 0$; if $j \equiv 1, 2(mod4)$
$= 1$; if $j \equiv 0, 3(mod4)$, $1 \leq j \leq k - 1$.

Case 4 $n \equiv 3(mod4)$.
Subcase I: $k \equiv 0(mod4)$.
$f(u_i) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $i \equiv 2, 3(mod4)$, $1 \leq i \leq n + 1$.
$f(v_i) = 0$; if $i \equiv 2, 3(mod4)$
$= 1$; if $i \equiv 0, 1(mod4)$, $1 \leq i \leq n + 1$.
$f(w_j) = 0$; if $j \equiv 1, 2(mod4)$
$= 1$; if $j \equiv 0, 3(mod4)$, $1 \leq j \leq k$.

Subcase II: $k \equiv 1(mod4)$.
$f(u_i) = 0$; if $i \equiv 0, 3(mod4)$
$= 1$; if $i \equiv 1, 2(mod4)$, $1 \leq i \leq n + 1$.
$f(v_i) = 0$; if $i \equiv 1, 2(mod4)$
$= 1$; if $i \equiv 0, 3(mod4)$, $1 \leq i \leq n + 1$.
$f(w_k) = 0$;
$f(w_j) = 0$; if $j \equiv 0, 3(mod4)$
$= 1$; if $j \equiv 1, 2(mod4)$, $1 \leq j \leq k - 1$.

Subcase III: $k \equiv 2(mod4)$.
$f(u_i) = 0$; if $i \equiv 0, 3(mod4)$
$= 1$; if $i \equiv 1, 2(mod4)$, $1 \leq i \leq n + 1$.
$f(v_i) = 0$; if $i \equiv 0, 3(mod4)$
$= 1$; if $i \equiv 1, 2(mod4)$, $1 \leq i \leq n + 1$.
$f(w_k) = 0$;
$f(w_j) = 0$; if $j \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4)$, $1 \leq j \leq k - 1$.

Subcase IV: $k \equiv 3(mod4)$.
$f(u_i) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $i \equiv 2, 3(mod4)$, $1 \leq i \leq n + 1$.
$f(v_i) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $i \equiv 2, 3(mod4)$, $1 \leq i \leq n + 1$.
$f(w_k) = 0$;
$f(w_j) = 0$; if $j \equiv 1, 2(mod4)$
$= 1$; if $j \equiv 0, 3(mod4)$, $1 \leq j \leq k - 1$.

The graph G under consideration satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ in each case which is shown in *Table 1*. Hence the graph G under consideration is cordial graph.

TABLE 1
Table for the graph G in *Theorem 2.1*.

Let $n = 4a + b, k = 4c + d,\ a, b, c, d \in N$.

b	d	vertex conditions	edge conditions
0	0,2	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1) + 1$
	1	$v_f(0) + 1 = v_f(1)$	$e_f(0) = e_f(1)$
	3	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$
1	0,2	$v_f(0) = v_f(1)$	$e_f(0) + 1 = e_f(1)$
	1	$v_f(0) + 1 = v_f(1)$	$e_f(0) = e_f(1)$
	3	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$
2	0,2	$v_f(0) = v_f(1)$	$e_f(0) + 1 = e_f(1)$
	1	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$
	3	$v_f(0) + 1 = v_f(1)$	$e_f(0) = e_f(1)$
3	0,2	$v_f(0) = v_f(1)$	$e_f(0) + 1 = e_f(1)$
	1,3	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$

Illustration 2.1 For better understanding of labeling pattern defined in *Theorem 2.1*, cordial labeling of the graph G obtained by joining two copies of fan graph F_4 by a path P_3 is shown in *Fig. 1*. It is the case related to $n \equiv 0(mod4)$ and $k \equiv 3(mod4)$.

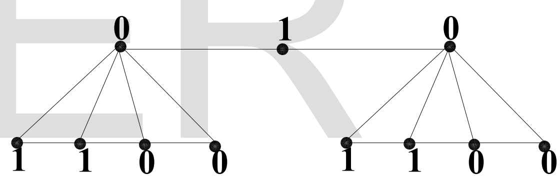

Fig. 1 : Cordial labeling of the graph G obtained by joining two copies of fan graph F_4 by path P_3.

Theorem 2.2 Star of fan graph F_n^* is cordial.
Proof: Let $v_1, v_2, \ldots, v_{n+1}$ be the the successive vertices of central copy of fan graph (where v_1 is apex vertex). Let $u_{i1}, u_{i2}, \ldots, u_{i(n+1)}$ be the successive vertices of i^{th} copy $F_n^{(i)}$ of fan graph except the central copy (where u_{i1} is apex vertex, $i = 1, 2 \ldots, n + 1$).
Let $e_i = v_i u_{i1}$ be the edge joining central copy and i^{th} copy of fan graph.

To define the required labeling function $f : V(F_n^*) \rightarrow \{0, 1\}$, we consider the following cases:
Case 1: $n \equiv 0(mod4)$.
In this case ,we define labeling as follows :
$f(v_i) = 0$; if $i \equiv 2, 3(mod4)$
$= 1$; if $i \equiv 0, 1(mod4)$, $1 \leq i \leq n + 1$.
When $i \equiv 2, 3(mod4)$,
$f(u_{ij}) = 0$; if $j \equiv 2, 3(mod4)$
$= 1$; if $j \equiv 0, 1(mod4)$, $1 \leq j \leq n + 1$.
When $i \equiv 0, 1(mod4)$,
$f(u_{ij}) = 0$; if $j \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4)$, $1 \leq j \leq n + 1$.

Case 2: $n \equiv 1(mod4)$.
In this case ,we define labeling as follows :
$f(v_i) = 0$; if $i \equiv 2,3(mod4)$
$= 1$; if $i \equiv 0,1(mod4)$, $1 \leq i \leq n+1$.
When $i \equiv 2,3(mod4)$,
$f(u_{ij}) = 0$; if $j \equiv 0,1(mod4)$,
$= 1$; if $j \equiv 2,3(mod4)$, $1 \leq j \leq n+1$.
When $i \equiv 0,1(mod4)$,
$f(u_{ij}) = 0$; if $j \equiv 2,3(mod4)$
$= 1$; if $j \equiv 0,1(mod4)$, $1 \leq j \leq n+1$.

Case 3: $n \equiv 2(mod4)$.
In this case, we define labeling as follows :
$f(v_i) = 0$; if $i \equiv 0,3(mod4)$
$= 1$; if $i \equiv 1,2(mod4)$, $1 \leq i \leq n+1$.
When $i \equiv 0,3(mod4)$,
$f(u_{ij}) = 0$; if $j \equiv 0,1(mod4)$
$= 1$; if $j \equiv 2,3(mod4)$, $1 \leq j \leq n+1$.
When $i \equiv 1,2(mod4)$,
$f(u_{ij}) = 0$; if $j \equiv 2,3(mod4)$
$= 1$; if $j \equiv 0,1(mod4)$, $1 \leq j \leq n+1$.

Case 4: $n \equiv 3(mod4)$.
In this case, we define labeling as follows :
$f(v_i) = 0$; if $i \equiv 0,3(mod4)$
$= 1$; if $i \equiv 1,2(mod4)$, $1 \leq i \leq n+1$.
When $i \equiv 0,3(mod4)$,
$f(u_{ij}) = 0$; if $j \equiv 0,1(mod4)$
$= 1$; if $j \equiv 2,3(mod4)$, $1 \leq j \leq n+1$.
When $i \equiv 1,2(mod4)$,
$f(u_{ij}) = 0$; if $j \equiv 0,3(mod4)$
$= 1$; if $j \equiv 1,2(mod4)$, $1 \leq j \leq n+1$.

The graph G under consideration satisfies the condition $|v_f(0)-v_f(1)| \leq 1$ and $|e_f(0)-e_f(1)| \leq 1$ in each case which is shown in *Table 2*. Hence the star of fan graph F_n^* is cordial graph.

TABLE 2
Table for *Theorem 2.2*.

Let $n = 4a+b$, $a, b \in N$.

b	vertex conditions	edge conditions
0	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1)+1$
1,2,3	$v_f(0) = v_f(1)$	$e_f(0)+1 = e_f(1)$

Illustration 2.2 For better understanding of labeling pattern defined in *Theorem 2.2*, cordial labeling of the graph F_4^* is shown in *Fig. 2*. It is the case related to $n \equiv 0(mod4)$.

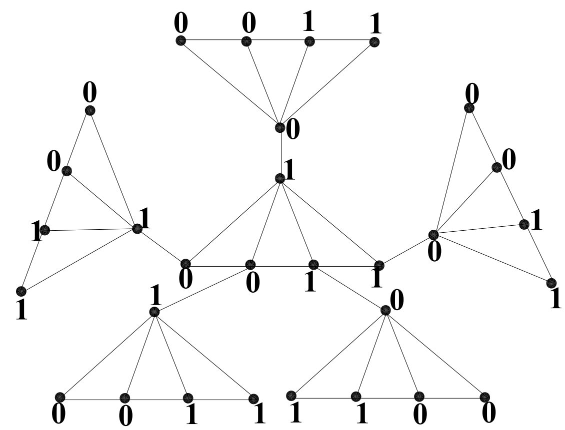

Fig. 2 : Cordial labeling of F_4^*.

Theorem 2.3 The graph obtained by joining two copies of star of fan graph F_n^* by a path of arbitrary length is cordial.

Proof: Let G be the graph obtained by joining two copies of F_n^* by path P_k.
Let us denote the successive vertices of central fan of the first copy of F_n^* by $v_1, v_2, \ldots, v_{n+1}$ (where v_1 is apex vertex) and the successive vertices of central fan of the second copy of F_n^* by $v'_1, v'_2, \ldots, v'_{n+1}$ (where v'_1 is apex vertex). Let $u_{i1}, u_{i2}, \ldots, u_{i(n+1)}$ be the successive vertices of i^{th} copy $F_n^{(i)}$ of first copy of F_n^* except the central copy (where u_{i1} is apex vertex, $i = 1, 2 \ldots, n+1$). Similarly let $u'_{i1}, u'_{i2}, \ldots, u'_{i(n+1)}$ be the successive vertices of i^{th} copy $F_n^{(i)}$ of second copy of F_n^* except the central copy (where u'_{i1} is apex vertex, $i = 1, 2 \ldots, n+1$).
Let $e_i = v_i u_{ij}$ be the edge joining the central copy of the graph with i^{th} copy of fan graph in the first copy of F_n^*. Similarly let $e'_i = v'_i u'_{ij}$ be the edge joining the central copy of the graph with i^{th} copy of fan graph in the second copy of F_n^*.
Let $w_1, w_2, \ldots, w_k$ be the vertices of path joining two copies of F_n^* with $w_1 = v_{n+1}$ and $w_k = v'_2$.
We define labeling function $f : V(G) \to \{0,1\}$ as follows.

Case 1: $n \equiv 0(mod4)$.
When $i \equiv 2,3(mod4)$,
$f(u_{ij}) = 0$; if $j \equiv 2,3(mod4)$
$= 1$; if $j \equiv 0,1(mod4)$, $1 \leq j \leq n+1$.
When $i \equiv 0,1(mod4)$,
$f(u_{ij}) = 0$; if $j \equiv 0,1(mod4)$
$= 1$; if $j \equiv 2,3(mod4)$, $1 \leq j \leq n+1$.

Subcase I: $k \equiv 0(mod4)$.
$f(v_i) = 0$; if $i \equiv 2,3(mod4)$
$= 1$; if $i \equiv 0,1(mod4)$, $1 \leq i \leq n+1$.
$f(v'_i) = 0$; if $i \equiv 0,3(mod4)$
$= 1$; if $i \equiv 1,2(mod4)$, $1 \leq i \leq n+1$.

International Journal of Scientific & Engineering Research, Volume 4, Issue 8 ISSN 2229-5518

When $i \equiv 0,3(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 2,3(mod4)$
$= 1$; if $j \equiv 0,1(mod4)$, $1 \leq i \leq n+1$.
When $i \equiv 1,2(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 0,1(mod4)$
$= 1$; if $j \equiv 2,3(mod4)$, $1 \leq i \leq n+1$.
$f(w_k) = 1$;
$f(w_t) = 0$; if $t \equiv 0,3(mod4)$
$= 1$; if $t \equiv 1,2(mod4)$, $1 \leq t \leq k-1$.

Subcase II: $k \equiv 1(mod4)$.
$f(v_i) = 0$; if $i \equiv 2,3(mod4)$
$= 1$; if $i \equiv 0,1(mod4)$, $1 \leq i \leq n+1$.
$f(v'_i) = 0$; if $i \equiv 2,3(mod4)$
$= 1$; if $i \equiv 0,1(mod4)$, $1 \leq i \leq n+1$.
When $i \equiv 2,3(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 2,3(mod4)$
$= 1$; if $j \equiv 0,1(mod4)$, $1 \leq i \leq n+1$.
When $i \equiv 0,1(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 0,1(mod4)$
$= 1$; if $j \equiv 2,3(mod4)$, $1 \leq i \leq n+1$.
$f(w_k) = 0$;
$f(w_t) = 0$; if $t \equiv 2,3(mod4)$
$= 1$; if $t \equiv 0,1(mod4)$, $1 \leq t \leq k-1$.

Subcase III: $k \equiv 2(mod4)$.
$f(v_i) = 0$; if $i \equiv 2,3(mod4)$
$= 1$; if $i \equiv 0,1(mod4)$, $1 \leq i \leq n+1$.
$f(v'_i) = 0$; if $i \equiv 2,3(mod4)$
$= 1$; if $i \equiv 0,1(mod4)$, $1 \leq i \leq n+1$.
When $i \equiv 2,3(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 2,3(mod4)$
$= 1$; if $j \equiv 0,1(mod4)$, $1 \leq i \leq n+1$.
When $i \equiv 0,1(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 0,1(mod4)$
$= 1$; if $j \equiv 2,3(mod4)$, $1 \leq i \leq n+1$.
$f(w_t) = 0$; if $t \equiv 2,3(mod4)$
$= 1$; if $t \equiv 0,1(mod4)$, $1 \leq t \leq k$.

Subcase IV: $k \equiv 3(mod4)$.
$f(v_i) = 0$; if $i \equiv 2,3(mod4)$
$= 1$; if $i \equiv 0,1(mod4)$, $1 \leq i \leq n+1$.
$f(v'_i) = 0$; if $i \equiv 0,3(mod4)$
$= 1$; if $i \equiv 1,2(mod4)$, $1 \leq i \leq n+1$.
When $i \equiv 0,3(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 2,3(mod4)$.
$= 1$; if $j \equiv 0,1(mod4)$, $1 \leq i \leq n+1$.
When $i \equiv 1,2(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 0,1(mod4)$.
$= 1$; if $i \equiv 2,3(mod4)$, $1 \leq i \leq n+1$.
$f(w_k) = 1$;
$f(w_t) = 0$; if $t \equiv 2,3(mod4)$
$= 1$; if $t \equiv 0,1(mod4)$, $1 \leq t \leq k-1$.

Case 2: $n \equiv 1(mod4)$.
When $i \equiv 2,3(mod4)$,
$f(u_{ij}) = 0$; if $j \equiv 0,1(mod4)$
$= 1$; if $j \equiv 2,3(mod4)$, $1 \leq j \leq n+1$.
When $i \equiv 0,1(mod4)$,
$f(u_{ij}) = 0$; if $j \equiv 2,3(mod4)$
$= 1$; if $j \equiv 0,1(mod4)$, $1 \leq j \leq n+1$.

Subcase I: $k \equiv 0(mod4)$.
$f(v_i) = 0$; if $i \equiv 2,3(mod4)$
$= 1$; if $i \equiv 0,1(mod4)$, $1 \leq i \leq n+1$.
$f(v'_i) = 0$; if $i \equiv 0,1(mod4)$
$= 1$; if $i \equiv 2,3(mod4)$, $1 \leq i \leq n+1$.
When $i \equiv 0,1(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 2,3(mod4)$
$= 1$; if $j \equiv 0,1(mod4)$, $1 \leq i \leq n+1$.
When $i \equiv 1,2(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 0,1(mod4)$
$= 1$; if $i \equiv 2,3(mod4)$, $1 \leq i \leq n+1$.
$f(w_t) = 0$; if $t \equiv 1,2(mod4)$
$= 1$; if $t \equiv 0,3(mod4)$, $1 \leq t \leq k$.

Subcase II: $k \equiv 1(mod4)$.
$f(v_i) = 0$; if $i \equiv 2,3(mod4)$
$= 1$; if $i \equiv 0,1(mod4)$, $1 \leq i \leq n+1$.
$f(v'_i) = 0$; if $i \equiv 0,1(mod4)$
$= 1$; if $i \equiv 2,3(mod4)$, $1 \leq i \leq n+1$.
When $i \equiv 0,1(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 2,3(mod4)$
$= 1$; if $j \equiv 0,1(mod4)$, $1 \leq j \leq n+1$.
When $i \equiv 1,2(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 0,1(mod4)$
$= 1$; if $i \equiv 2,3(mod4)$, $1 \leq j \leq n+1$.
$f(w_k) = 1$;
$f(w_t) = 0$; if $t \equiv 1,2(mod4)$
$= 1$; if $t \equiv 0,3(mod4)$, $1 \leq t \leq k-1$

Subcase III: $k \equiv 2(mod4)$.
$f(v_i) = 0$; if $i \equiv 2,3(mod4)$
$= 1$; if $i \equiv 0,1(mod4)$, $1 \leq i \leq n+1$.
$f(v'_i) = 0$; if $i \equiv 2,3(mod4)$
$= 1$; if $i \equiv 0,1(mod4)$, $1 \leq i \leq n+1$.
When $i \equiv 2,3(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 2,3(mod4)$
$= 1$; if $j \equiv 0,1(mod4)$, $1 \leq j \leq n+1$.
When $i \equiv 0,1(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 0,1(mod4)$
$= 1$; if $i \equiv 2,3(mod4)$, $1 \leq j \leq n+1$.
$f(w_t) = 0$; if $t \equiv 1,2(mod4)$
$= 1$; if $t \equiv 0,3(mod4)$, $1 \leq t \leq k$.

Subcase IV: $k \equiv 3(mod4)$.
$f(v_i) = 0$; if $i \equiv 2,3(mod4)$
$= 1$; if $i \equiv 0,1(mod4)$, $1 \leq i \leq n+1$.
$f(v'_i) = 0$; if $i \equiv 2,3(mod4)$
$= 1$; if $i \equiv 0,1(mod4)$, $1 \leq i \leq n+1$.
When $i \equiv 2,3(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 2,3(mod4)$

International Journal of Scientific & Engineering Research, Volume 4, Issue 8 ISSN 2229-5518

$= 1$; if $j \equiv 0,1(mod4)$, $1 \leq j \leq n+1$.
When $i \equiv 0,1(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 0,1(mod4)$
$= 1$; if $i \equiv 2,3(mod4)$, $1 \leq j \leq n+1$.
$f(w_k) = 0$;
$f(w_t) = 0$; if $t \equiv 1,2(mod4)$
$= 1$; if $t \equiv 0,3(mod4)$, $1 \leq t \leq k-1$.

Case 3: $n \equiv 2(mod4)$.
When $i \equiv 0,3(mod4)$,
$f(u_{ij}) = 0$; if $j \equiv 0,1(mod4)$
$= 1$; if $j \equiv 2,3(mod4)$, $1 \leq j \leq n+1$.
When $i \equiv 1,2(mod4)$,
$f(u_{ij}) = 0$; if $j \equiv 2,3(mod4)$
$= 1$; if $i \equiv 0,1(mod4)$, $1 \leq j \leq n+1$.

Subcase I: $k \equiv 0(mod4)$.
$f(v_i) = 0$; if $i \equiv 0,3(mod4)$
$= 1$; if $i \equiv 1,2(mod4)$, $1 \leq i \leq n+1$.
$f(v'_i) = 0$; if $i \equiv 0,3(mod4)$
$= 1$; if $i \equiv 1,2(mod4)$, $1 \leq i \leq n+1$.
When $i \equiv 0,3(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 0,1(mod4)$
$= 1$; if $j \equiv 2,3(mod4)$, $1 \leq j \leq n+1$.
When $i \equiv 1,2(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 2,3(mod4)$
$= 1$; if $i \equiv 0,1(mod4)$, $1 \leq j \leq n+1$.
$f(w_t) = 0$; if $t \equiv 1,2(mod4)$
$= 1$; if $t \equiv 0,3(mod4)$, $1 \leq t \leq k$.

Subcase II: $k \equiv 1(mod4)$.
$f(v_i) = 0$; if $i \equiv 0,3(mod4)$
$= 1$; if $i \equiv 1,2(mod4)$, $1 \leq i \leq n+1$.
$f(v'_i) = 0$; if $i \equiv 0,3(mod4)$
$= 1$; if $i \equiv 1,2(mod4)$, $1 \leq i \leq n+1$.
When $i \equiv 0,3(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 0,1(mod4)$
$= 1$; if $j \equiv 2,3(mod4)$, $1 \leq j \leq n+1$.
When $i \equiv 1,2(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 2,3(mod4)$
$= 1$; if $i \equiv 0,1(mod4)$, $1 \leq j \leq n+1$.
$f(w_k) = 1$;
$f(w_t) = 0$; if $t \equiv 1,2(mod4)$
$= 1$; if $t \equiv 0,3(mod4)$, $1 \leq t \leq k-1$.

Subcase III: $k \equiv 2(mod4)$.
$f(v_i) = 0$; if $i \equiv 0,3(mod4)$
$= 1$; if $i \equiv 1,2(mod4)$, $1 \leq i \leq n+1$.
$f(v'_{n+1}) = 1$;
$f(v'_i) = 0$; if $i \equiv 2,3(mod4)$
$= 1$; if $i \equiv 0,1(mod4)$, $1 \leq i \leq n$.
When $i \equiv 2,3(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 0,1(mod4)$
$= 1$; if $j \equiv 2,3(mod4)$, $1 \leq j \leq n+1$.
When $i \equiv 0,1(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 2,3(mod4)$
$= 1$; if $i \equiv 0,1(mod4)$, $1 \leq j \leq n+1$.
$f(w_t) = 0$; if $t \equiv 1,2(mod4)$
$= 1$; if $t \equiv 0,3(mod4)$, $1 \leq t \leq k$.

Subcase IV: $k \equiv 3(mod4)$.
$f(v_i) = 0$; if $i \equiv 0,3(mod4)$
$= 1$; if $i \equiv 1,2(mod4)$, $1 \leq i \leq n+1$.
$f(v'_{n+1}) = 1$;
$f(v'_i) = 0$; if $i \equiv 2,3(mod4)$
$= 1$; if $i \equiv 0,1(mod4)$, $1 \leq i \leq n$.
When $i \equiv 2,3(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 0,1(mod4)$
$= 1$; if $j \equiv 2,3(mod4)$, $1 \leq j \leq n+1$.
When $i \equiv 0,1(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 2,3(mod4)$
$= 1$; if $i \equiv 0,1(mod4)$, $1 \leq j \leq n+1$.
$f(w_k) = 0$;
$f(w_t) = 0$; if $t \equiv 1,2(mod4)$
$= 1$; if $t \equiv 0,3(mod4)$, $1 \leq t \leq k-1$.

Case 4: $n \equiv 3(mod4)$.
When $i \equiv 0,3(mod4)$,
$f(u_{ij}) = 0$; if $j \equiv 0,1(mod4)$
$= 1$; if $j \equiv 2,3(mod4)$, $1 \leq j \leq n+1$.
When $i \equiv 1,2(mod4)$,
$f(u_{ij}) = 0$; if $j \equiv 0,3(mod4)$
$= 1$; if $i \equiv 1,2(mod4)$, $1 \leq j \leq n+1$.

Subcase I: $k \equiv 0(mod4)$.
$f(v_i) = 0$; if $i \equiv 0,3(mod4)$
$= 1$; if $i \equiv 1,2(mod4)$, $1 \leq i \leq n+1$.
$f(v'_i) = 0$; if $i \equiv 0,3(mod4)$
$= 1$; if $i \equiv 1,2(mod4)$, $1 \leq i \leq n+1$.
When $i \equiv 0,3(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 0,1(mod4)$
$= 1$; if $j \equiv 2,3(mod4)$, $1 \leq j \leq n+1$.
When $i \equiv 1,2(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 0,3(mod4)$
$= 1$; if $i \equiv 1,2(mod4)$, $1 \leq j \leq n+1$.
$f(w_t) = 0$; if $t \equiv 1,2(mod4)$
$= 1$; if $t \equiv 0,3(mod4)$, $1 \leq t \leq k$.

Subcase II: $k \equiv 1(mod4)$.
$f(v_i) = 0$; if $i \equiv 0,3(mod4)$
$= 1$; if $i \equiv 1,2(mod4)$, $1 \leq i \leq n+1$.
$f(v'_i) = 0$; if $i \equiv 0,3(mod4)$
$= 1$; if $i \equiv 1,2(mod4)$, $1 \leq i \leq n+1$.
When $i \equiv 0,3(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 0,1(mod4)$
$= 1$; if $j \equiv 2,3(mod4)$, $1 \leq j \leq n+1$.
When $i \equiv 1,2(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 0,3(mod4)$
$= 1$; if $i \equiv 1,2(mod4)$, $1 \leq j \leq n+1$.
$f(w_k) = 1$;
$f(w_t) = 0$; if $t \equiv 1,2(mod4)$
$= 1$; if $t \equiv 0,3(mod4)$, $1 \leq t \leq k-1$.

Subcase III: $k \equiv 2(mod4)$.
$f(v_i) = 0$; if $i \equiv 0, 3(mod4)$
$= 1$; if $i \equiv 1, 2(mod4)$, $1 \leq i \leq n + 1$.
$f(v'_i) = 0$; if $i \equiv 2, 3(mod4)$
$= 1$; if $i \equiv 0, 1(mod4)$, $1 \leq i \leq n + 1$.
When $i \equiv 2, 3(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4)$, $1 \leq j \leq n + 1$.
When $i \equiv 0, 1(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 0, 3(mod4)$
$= 1$; if $i \equiv 1, 2(mod4)$, $1 \leq j \leq n + 1$.
$f(w_t) = 0$; if $t \equiv 1, 2(mod4)$
$= 1$; if $t \equiv 0, 3(mod4)$, $1 \leq t \leq k$.

Subcase IV: $k \equiv 3(mod4)$.
$f(v_i) = 0$; if $i \equiv 0, 3(mod4)$
$= 1$; if $i \equiv 1, 2(mod4)$, $1 \leq i \leq n + 1$.
$f(v'_i) = 0$; if $i \equiv 2, 3(mod4)$
$= 1$; if $i \equiv 0, 1(mod4)$, $1 \leq i \leq n + 1$.
When $i \equiv 2, 3(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 0, 1(mod4)$
$= 1$; if $j \equiv 2, 3(mod4)$, $1 \leq j \leq n + 1$.
When $i \equiv 0, 1(mod4)$,
$f(u'_{ij}) = 0$; if $j \equiv 0, 3(mod4)$
$= 1$; if $i \equiv 1, 2(mod4)$, $1 \leq j \leq n + 1$.
$f(w_k) = 0$;
$f(w_t) = 0$; if $t \equiv 1, 2(mod4)$
$= 1$; if $t \equiv 0, 3(mod4)$, $1 \leq t \leq k - 1$.

The graph G under consideration satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ in each case which is shown in *Table 3*. Hence the graph G is cordial.

TABLE 3
Table for the graph G in *Theorem 2.3*.

Let $n = 4a + b, k = 4c + d$, $a, b, c, d \in N$.

b	d	vertex conditions	edge conditions
0	0	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1) + 1$
	1	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$
	2	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1) + 1$
	3	$v_f(0) + 1 = v_f(1)$	$e_f(0) = e_f(1)$
1	0,2	$v_f(0) = v_f(1)$	$e_f(0) + 1 = e_f(1)$
	1,3	$v_f(0) + 1 = v_f(1)$	$e_f(0) = e_f(1)$
2,3	0,2	$v_f(0) = v_f(1)$	$e_f(0) + 1 = e_f(1)$
	1	$v_f(0) + 1 = v_f(1)$	$e_f(0) = e_f(1)$
	3	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$

Illustration 2.3 For better understanding of labeling pattern defined in *Theorem 2.3* cordial labeling of the graph G obtained by joining two .3.ies of star of fan graphs F_3^* by a path P_4 is shown in *Fig. 3*. It is the case related to $n \equiv 3(mod4)$ and $k \equiv 0(mod4)$.

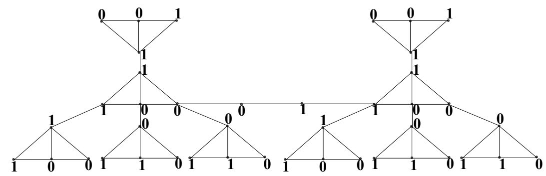

Fig. 3 : Cordial labeling of the graph G obtained by joining two copies of F_3^* by path P_4.

3 Concluding Remarks

The research work carried out in this paper is novel and it contributes three new graphs in the theory of cordial graphs. The labeling pattern is given in detail and it is shown by sufficient illustrations. The table helps to understand condition of cordiality in each of the theorem.

4 Scope of Further Research

In this paper cordiality of star of fan graph is discussed. Also it is investigated that the graph obtained by joining two copies of star of fan graph by a path of arbitrary length is also cordial. This result may be extended by taking arbitrary number of copies of star of fans. Here we also proved that the graph obtained by joining two copies of fan graphs by a path of arbitrary length is cordial. This result may be extended by taking arbitrary number of copies of fan graph. Moreover all above results can be discussed for 3 equitable labeling.

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